compute. We see that it is possible to extract the richness of detailed information inherent in a time-dependent, multicomponent system by properly analyzing the preserved time record of a multiplexed data processing network. The cost will be some degradation in the S/N ratio. The extent of this degradation will depend on the quality of the multiplexed data processing system. In general, such a system will increase S/N ratio requirements. A minor consideration is that each of the stepped runs must be maintained for a time T, which is long compared to any characteristic time of the system under study. Finally, we note that this analysis may be used in conjunction with modulation spectroscopy to minimize background noise in the acquisition of a spectrum.

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Compatibility Conditions from Deformation Displacement Relationship

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Nomenclature

$[B]$	= equilibrium matrix
$[C]$	= compatibility matrix
$\{F\}$	= internal force vector
$[G]$	= concatenated flexibility matrix
IFM	= integrated force method
$[J]$	= displacement coefficient matrix
$\{p\}$	= external load vector
$[S]$	= IFM transformation matrix
SFM	= standard force method
(u,v)	= displacement components
$\{X\}$	= displacement vector
$\{\beta\}$	= deformation vector
$(\epsilon_x,\epsilon_y,\gamma_{xy})$	= plane strain components

Introduction

IN the novel formulation termed the integrated force method (IFM),[1-3] the compatibility conditions (CC) are generated following St. Venant's procedure.[4] In St. Venant's procedure of elasticity, the compatibility conditions are obtained by eliminating the displacements from the strain displacement relationships (SDR). The St. Venant's procedure has now been extended to discrete analysis, and the CC in the IFM are generated by eliminating the displacements from the deformation displacement relationship (DDR). The compatibility conditions thus generated are banded, and the bandwidth is obtained from the element numbering of the discretization.

Basic Theory of Integrated Force Method

In the IFM for discrete analyses, a structure is symbolized as "structure (m,n)," where m is the displacement degrees of freedom (dof) and n the force degrees of freedom (fof). These $(m+n)$ variables must satisfy the following m equilibrium and $(n-m)$ compatibility equation as

Equilibrium equations (EE):

$$[B]\{F\}=\{p\} \tag{1}$$

Compatibility conditions (CC):

$$[C]\{\beta\}=\{0\} \tag{2}$$

The matrices $[B]$ and $[C]$ are independent of design parameters.

In the IFM, Eq. (1) is retained in its original form; however, the deformations $\{\beta\}$ are transformed to forces as $\{\beta\}=[G]\{F\}$. Thus Eq. (2) in forces can be rewritten as

$$[C][G]\{F\}=\{0\} \tag{3}$$

Coupling Eq. (1) to Eq. (3) yields the IFM as

$$\begin{bmatrix} [B] \\ \hdashline [C]\,[G] \end{bmatrix}\{F\}=\begin{Bmatrix} p \\ \hdashline 0 \end{Bmatrix} \quad \text{or} \quad [S]\{F\}=\{p\}^* \tag{4}$$

The displacements $\{X\}$ are generated from the forces $\{F\}$ using the following formula,[2,3]

$$\{X\}=[J][G]\{F\} \tag{5}$$

Compatibility Matrix from Deformation Displacement Relationship

St. Venant's formulation of the CC is illustrated taking the example of plane elasticity problem. For the problem, the SDR can be written as

$$\epsilon_x=\frac{\partial u}{\partial x}; \qquad \epsilon_y=\frac{\partial v}{\partial y}, \qquad \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \tag{6}$$

Elimination of displacements from the SDR yield the CC as

$$\frac{\partial^2\epsilon_x}{\partial y^2}+\frac{\partial^2\epsilon_y}{\partial x^2}-\frac{\partial^2\gamma_{xy}}{\partial x\partial y}=0 \tag{7}$$

The two steps of St. Venant's procedure are: 1) establish the SDR and 2) eliminate the displacements from the SDR to obtain the compatibility conditions. The DDR for discrete analysis has been formulated on an energy basis. The internal energy (IE) of structure (m,n) can be written as

$$\text{IE}=\tfrac{1}{2}\{F\}^T\{\beta\}=\tfrac{1}{2}\{X\}^T\{p\}=\tfrac{1}{2}\{X\}^T[B]\{F\} \tag{8}$$

or

$$\tfrac{1}{2}\{F\}^T\Big[[B]^T\{X\}-\{\beta\}\Big]=0$$

Since the force vector is not null, the DDR can be written as

$$\{\beta\}=[B]^T\{X\} \tag{9}$$

Elimination of m displacements from the n DDR given by Eq. (8) yields the $r=(n-m)$ compatibility conditions and the associated coefficient matrix $[C]$.

Bandwidth of the Compatibility Matrix

In the finite-element analysis, the elemental deformations have to be compatible with the neighboring elements; thus the CC are banded. The maximum bandwidth (MBW) of the

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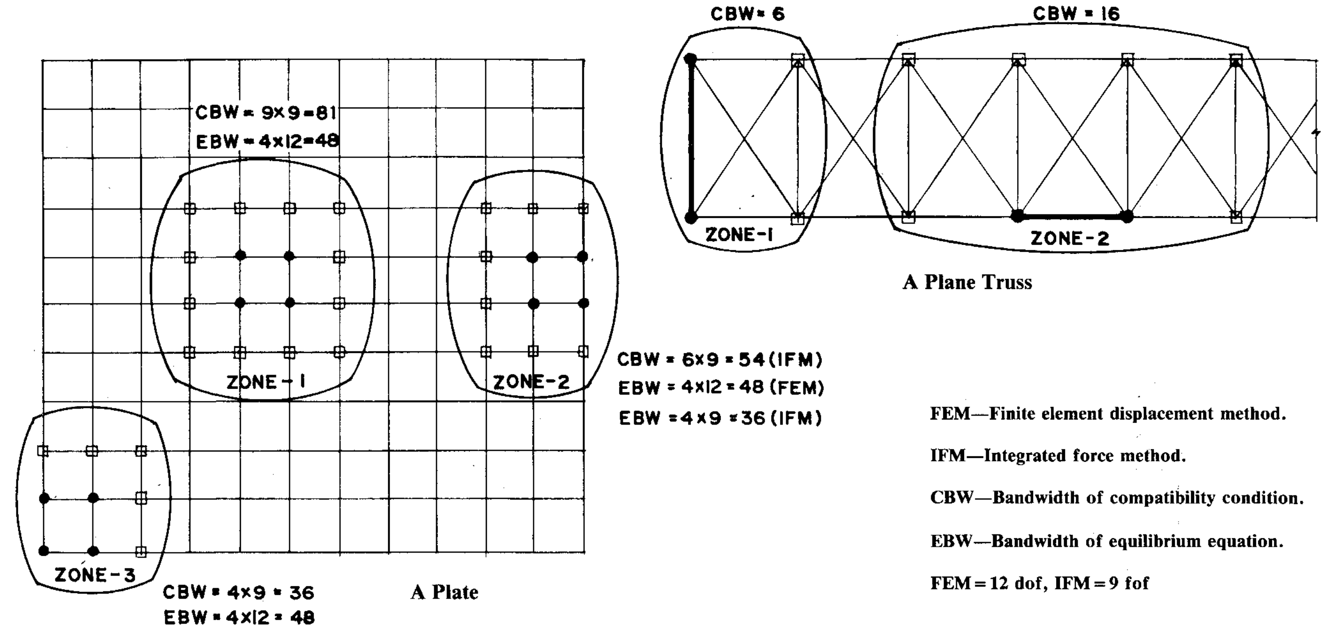

Fig. 1 Bandwidth of compatibility conditions.

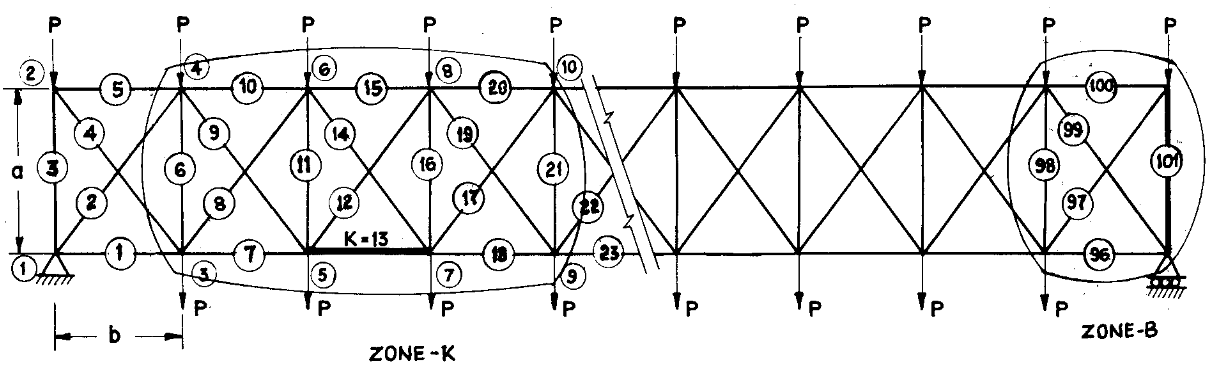

Fig. 2 20-bay truss.

Table 1 Comparison of the properties of the compatibility matrices

Structure types	Matrix bandwidth				Sparsity ratio	
	Average		Maximum		Compatibility matrix $[C]$	Equilibrium matrix $[B]$
	Proposed	Ref. 8	Proposed	Ref. 8		
Truss (8,10)	6	6	6	6	0.545	0.242
Frame (44,81)	13.88	15.05	16	21	0.123	0.155
Plate (59,144)	22.38	35	38	39	0.0408	0.0425

CC of an element depends on the force degrees of freedom of its neighboring elements. For illustration we take the examples of a plate and a truss shown in Fig. 1. The force degrees of freedom (fof) for the plate element is $\text{fof}_p = 9$ and truss element is $\text{fof}_t = 1$. The MBWs of the compatibility matrix $[C]$ are as follows.

Plate: interior element $(\text{MBW})_{\text{Zone 1}} = 81$; boundary (element)$_{\text{Zone 2}} = 54$; and corner element $(\text{MBW})_{\text{Zone 3}} = 36$.

Truss: interior element $(\text{MBW})_{\text{Zone 1}} = 16$ and boundary element $(\text{MBW})_{\text{Zone 2}} = 6$.

Numerical Illustration

In the literature, there are four schemes[5-8] to generate the CC. Among them, the "turn-back LU"[7] is perhaps by far the best procedure as far as sparsity and bandwidth of the matrix $[C]$ are concerned. The present procedure is compared to the "turn-back LU" for three examples: a truss (8,10), a frame (44,81), and a plate (59,144). The configurations of the three structures depicted in Ref. 8 are not repeated here. The bandwidths of the CC for the examples as obtained by the two procedures are presented in Table 1, and it is observed that for the simple truss (8,10) both procedures yield an identical compatibility matrix. However, for both the frame and the plate the matrices from DDR have smaller bandwidths than those obtained by the turn-back LU.[7]

Two other examples, a truss and a plate, as shown in Figs. 2 and 3, are considered for generation of the compatibility matrix $[C]$ from DDR. The sparsity ratio and bandwidths of the matrices are presented in Table 2, and the following observations are arrived at from the examples solved.

1) The compatibility conditions are banded. The average bandwidth of the CC is smaller than that of equilibrium equations. For example, for the truss and plate, these are (8,6) and (17.8,8.93), respectively.

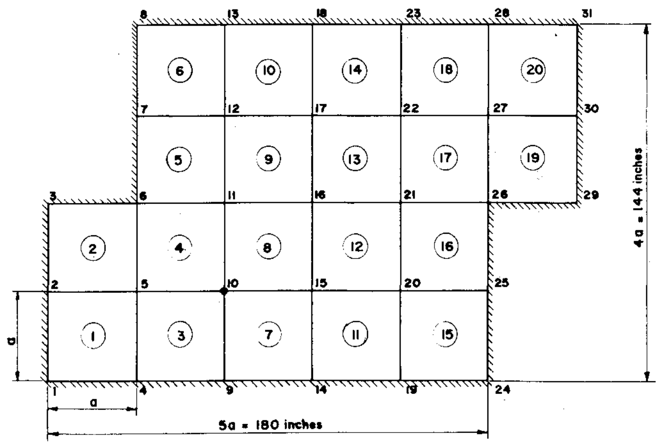

Fig. 3 Plate with fixed boundaries.

Table 2 Properties of compatibility matrices by DDR

	Equilibrium matrix $[B]$		Compatibility matrix $[C]$	
Parameter	Truss (81,101)	Plate (33,180)	Truss (81,101)	Plate (33,180)
Maximum bandwidth	8	18	6	23
Average bandwidth	8	17.18	6	8.93
Percentage sparsity	4.82	5.85	5.94	2.04

2) The sparsity ratios of the compatibility matrix $[C]$ are comparable to the equilibrium matrix for frame works. For plates, the sparsity ratio of compatibility matrix is smaller than the equilibrium matrix (2.04,5.85).

Conclusions

1) In the IFM the compatibility conditions are generated from the deformation displacement relations of the structure without any reference to the popular redundants or basis determinate structure of the SFM.

2) The upper bound of the bandwidth of the compatibility condition depends on the element numbering of the discretization.

3) The bandwidth and sparsity ratio of the compatibility matrix $[C]$ are comparable to those of the equilibrium matrix $[B]$ for frame structures.

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Thermal Effect on Frequencies of Coupled Vibrations of Pretwisted Rotating Beams

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Introduction

IN recent years, interest in the effect of temperature on solid bodies has increased greatly because of rapid developments in space technology, high-speed atmospheric flights, and nuclear energy applications. Sufficient work is available on coupled vibrations of beams.[1-5] It is well known[6] that, in the presence of a constant thermal gradient, the elastic coefficients of homogeneous materials become functions of the space variables. Fauconneau and Marangoni[7] have investigated the effect of the nonhomogeneity caused by a thermal gradient on the natural frequencies of simply supported plates of uniform thickness. Recently, Tomar and Jain[8] have studied the thermal effect on frequencies of a rotating wedge-shaped beam.

The purpose of this Note is to study the effect of a constant thermal gradient on coupled bending-bending-torsional vibrations of a pretwisted slender beam (that could represent a turbine blade of simple geometry) attached to a disk of radius r_0 as the disk rotates with an angular velocity Ω. A method based on Rayleigh's quotient is used to obtain upper bounds of the frequencies corresponding to the first three vibration modes. The frequencies for various values of angle of pretwist, hub-radius change, and the temperature gradient are obtained for setting the angle $\pi/2$.

Analysis and Equation of Motion

It is assumed that the beam is subjected to a steady one-dimensional temperature distribution along the length, i.e., in the z direction.

$$T = T_0(1-\xi) \tag{1}$$

where T denotes the temperature excess above the reference temperature at any point at a distance $\xi = z/L$ and T_0 denotes the temperature excess above the reference temperature at the end $z = L$ or $\xi = 1$.

The temperature dependence of the modulus of elasticity for most engineering material is given by

$$E(T) = E_1(1-\gamma T) \tag{2}$$

where E_1 is the value of the modulus of elasticity at the reference temperature, i.e., at $T=0$ along the z direction. Taking the temperature at the end of the beam, i.e., at $\xi = 1$, as the reference temperature, the modulus variation becomes

$$E(\xi) = E_1\{1-\alpha(1-\xi)\} \tag{3}$$

where the temperature gradient $\alpha = \gamma T_0$ $(0 \le \alpha < 1)$.

The differential equations for coupled bending-bending-torsional vibrations of a pretwisted rotating beam are

$$\frac{\partial^2}{\partial z^2}\left(EI'_{yy}\frac{\partial^2 u}{\partial z^2} + EI'_{xy}\frac{\partial^2 v}{\partial z^2}\right) = -\rho A\frac{\partial^2}{\partial t^2}(u+\delta_y\theta) + \frac{\partial^2 M_x}{\partial z^2} \tag{4a}$$

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